United States Naval Postgraduate School

THESIS

OPTIMAL ALLOCATION OF

PACIFIC FLEET PATROL AIRCRAFT

AMONG SELECTED DEPLOYMENT SITES

by

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Optimal Allocation of

Pacific Fleet Patrol Aircraft

Among Selected Deployment Sites

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ABSTRACT

A methodology is developed which determines the optimal allocation of patrol forces among selected deployment sites. The procedure uses a linear programming algorithm which minimizes a linear cost function, subject to restraining equations representing the total hours available, the relationship between on-station and transit hours, and base loading. A computer program is presented which translates input data into the format required by the IBM Mathematical Programming System/360 for the problem solution. The methodology can be utilized to determine the allocation of forces among selected bases, reallocation of forces when a base or bases must be removed from consideration, and the effect of utilizing additional bases.

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LIST OF SYMBOLS

(i, j)	= subarea formed by intersection of i^{th} row and j^{th} column
XL	= length of side of subarea parallel to x-axis
YL	= length of side of subarea parallel to y-axis
(x_k, y_k)	= location of base k
R_{ijk}	= distance from base k to area (i, j)
T_{os_k}	= available on-station time from base k
T_k	= average sortie length from base k
CFH_k	= cost per flight-hour when flown from base k
C_{ijk}	= cost per on-station hour in area (i, j) when flown from base k
b_{ij}	= on-station time required in area (i, j)
X_{ijk}	= on-station hours allocated to area (i, j) from base k
$x_{m+i,jk}$	= transit hours flown between area (i, j) and base k
x_{mis}	= total hours available for training and miscellaneous flying activity
a_k	= flight time available from base k
A	= total flight-hours available

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I. INTRODUCTION

A. BACKGROUND

At the present time the deployment concepts associated with the Navy's patrol aircraft in the Pacific Theater are little removed from those which evolved following the close of World War II. A majority of the advance bases currently supporting U. S. Naval Forces in the Western Pacific were acquired during the years following the Second World War. At that time the predominate thought concerning the positioning of advance forces was that the first line of defense should be as far away from the continental United States as possible. Covering nearly all of the transit routes between the Asian mainland and the Central Pacific, this chain of bases has provided the United States with a convenient surveillance platform.

As long as the continued presence of the United States is required in the Western Pacific to protect U. S. interests, the Navy must be ready to provide adequate forces for the following:

1. Control of the sea-lanes and sea-areas against threats to United States interests, forces or commitments.

2. Continuing peacetime deployments in order to deter aggression and to support United States policy as it may evolve.

3. Special surveillance, intelligence, and counter-surveillance operations.

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It may be assumed that due to U. S. commitments established under the United Nations Charter, participation in SEATO and the ANZUS agreement, and many bi-lateral agreements and assurances that the advanced deployment of U. S. Naval Forces in the Western Pacific will be required into the 1970's.

Since naval forces are to be deployed during the next several years in approximately the same areas where they have been deployed over the past 10 years, the existing base structure may be regarded as adequate. It would be difficult to improve the geographical positioning of the present base structure without moving onto the Asian mainland, which is an alternative many military planners do not wish to consider.

While the commitment of U. S. forces overseas is very likely to continue at or near its present level for the next few years, the continued use of all present bases for the same time span is in considerable doubt. It is entirely possible that continuing political pressure by groups in host countries may result in the denial of some bases to U. S. forces; for example, the Status of Forces Agreement with Japan is up for optional termination after 1970 on twelve months notice.

Thought has already been given to a retrenchment to Guam, the only base site in the Western Pacific to which the U. S. has continuing access, and to the Micronesian Islands, which the U. S. holds under a United Nations trusteeship. Called a "strategic trusteeship," it allows the U. S. to erect fortifications and garrison troops on the islands.

B. OBJECTIVE

The increasing possibility of base denial and the rising cost of operating and equipping overseas forces have brought about the need for a reappraisal of present deployment concepts and the development of a method for the optimal allocation of available forces among available bases.

It is the purpose of this thesis to present a method with which operational commanders may optimally allocate the patrol forces at their disposal, subject to operational requirements, operating areas, and forces available.

The procedure developed requires as input ~~data, information~~ concerning the location of existing bases, the desired coverage of surveillance areas, and the amount of flight time available. Utilizing the Mathematical Programming System/360 Linear Programming package (MPS/360 LP), available for the IBM 360 computers, a solution is determined which provides a minimum cost allocation of flight-hours among participating bases.

The number of aircraft required at each location may be determined by comparing the number of flight-hours required with the flying hour capability of the aircraft. Since it is unlikely that this comparison will result in an integer solution for the number of aircraft required, it is necessary to round off to the next higher integer value. This will generally result in additional flight-hours being made available for training flights and other uses. Appendix B, combining the methods of

Sunde [1] and Mooz [2], presents a formulation for determining the flying hour capability of an aircraft from a knowledge of its operating hours and available maintenance data.

C. ORGANIZATION

In the formulation of this methodology the basic system considered is the P-3 series land based patrol aircraft and its supporting bases. No distinction is made between the various models of the basic P-3 aircraft.

A brief description of this aircraft, its operating characteristics, capabilities, and requirements is contained in Section II of this thesis. Also contained in Section II is a listing of some of the overseas bases capable of supporting P-3 operations.

Section III presents the development of the methodology. A general linear programming formulation is followed in which a linear objective function denoting cost is minimized subject to a series of constraining relationships.

Section IV discusses possible extensions of the methodology, inadequacies of some of the assumptions, and areas in need of further study. The thesis concludes with Section V, which presents a summary of the development.

Three appendices, A, B, and C, provide supplementary information. Appendix A contains the development of a linear approximation of the relationship between operating radius and on-station time.[1] Appendix B presents a method of determining the maximum flight-hour capability of an aircraft from available operational and maintenance data. In Appendix C, a sample problem is solved to demonstrate the use of the methodology. Also presented is the computer program, written in FORTRAN IV, which converts the input data for a problem into the format required for input into the linear programming algorithm.

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[1]On-station time is defined to be that time spent in a specific operating area and does not include time necessary to transit to and from the operating area.

II. SYSTEM DESCRIPTION

The system referred to in the section heading is considered to mean the P-3 series aircraft and its supporting bases. Although some earlier P-2 series aircraft are still in use, the fleetwide transition to the P-3 is sufficiently well along that only the P-3 will be considered in this thesis.

A. AIRCRAFT

The P-3 is a four-engine, low-wing, all-weather aircraft designed for patrol operations and antisubmarine warfare. It is in the 127,000-pound gross weight class and is powered by four turboprop engines. The aircraft is fully pressurized and is capable of operating at all altitudes from Sea Level up to 34,000 feet and at speeds of from 150 to 400 knots. As presented in Appendix A, during a normal mission time of 11.2 to 12.0 hours, the P-3 can transit to an operating area at a distance of over 1300 nautical miles and remain on-station for a period of four hours.

The aircraft is normally manned by a crew of 12 men consisting of a pilot, copilot, navigator, tactical coordinator, flight engineer, and six technical specialists.

Under normal operating conditions the aircraft will fly "profile" missions. Utilizing this "profile" concept, the aircraft will transit to a patrol area at altitudes between 17,000 feet and 22,000 feet at a speed of 300-330 knots. The enroute altitude will generally depend upon the wind at different altitudes, distance to operating area, and takeoff weight. Upon arrival in the operating area, the aircraft descends to search altitude and reduces to maximum endurance airspeed. It is during this on-station period that one or possibly two of the aircraft's engines may be "feathered"[2] to increase the available on-station time. The return trip is usually made at a altitude of 25,000 feet to 30,000 feet.

B. BASES

By considering the operating requirements of the P-3, the takeoff and landing distances, the fuel required, the necessary personnel, and the aircraft support requirements--and by referring to a listing of the major aerodromes is the Western Pacific, it is possible to compile a list of feasible operating bases for the P-3 aircraft. Table I presents a listing of bases which might be selected.

Utilizing Table I and the information on operating radius versus on-station time as presented in Appendix A, Figure 1 may be drawn. From Figure 1 it can be observed that the P-3 aircraft, operating from

[2] A feathered engine, in this case, refers to one which has been shut down by the pilot to conserve fuel but which may be started at a later time.

suitable bases, can provide at least four hours of on-station coverage over a majority of the ocean area of the Western Pacific. It should be noted that in many areas a significant amount of overlap is provided. It is the optimal coverage of these areas of overlay which the methodology seeks to provide.

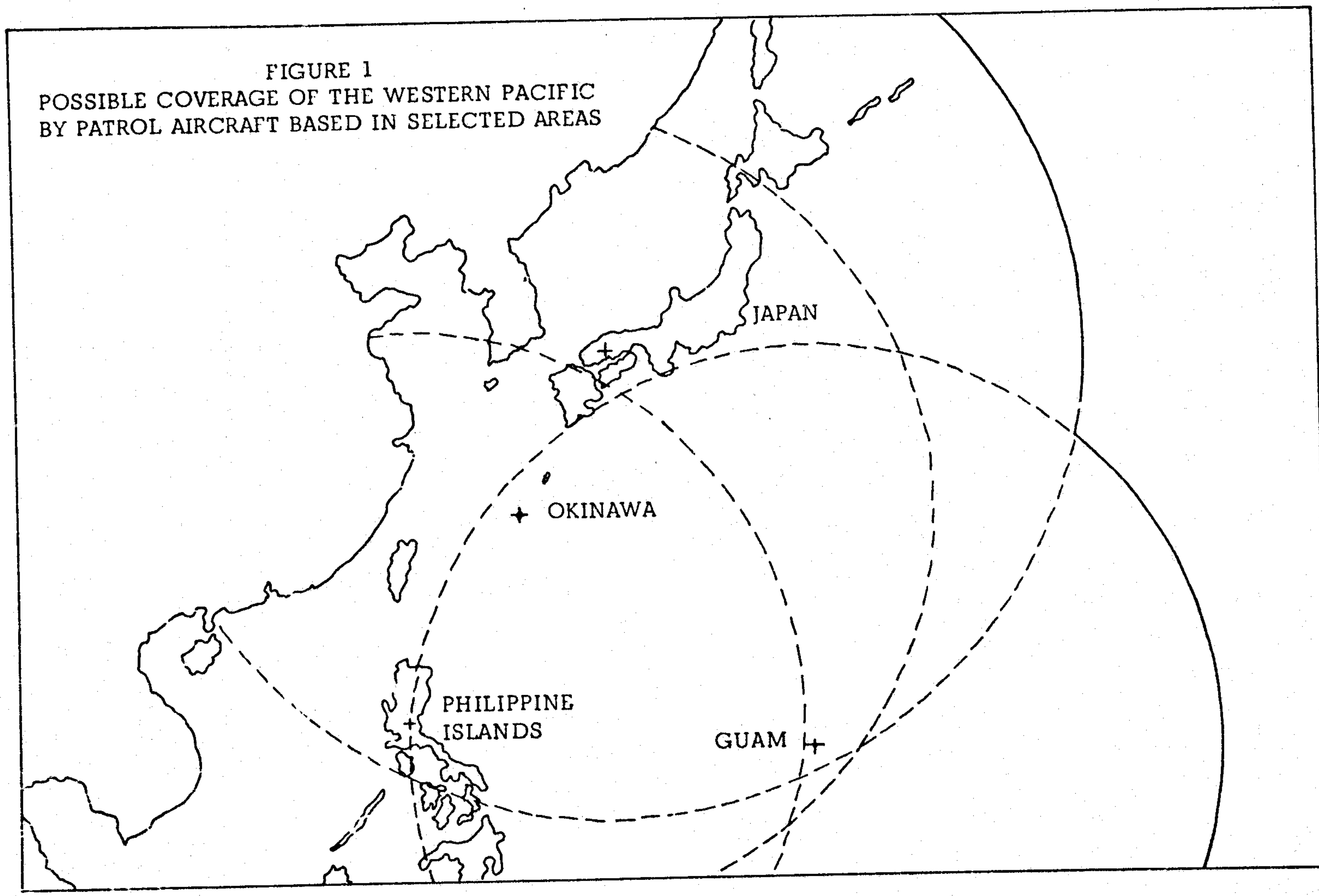
TABLE I

AERODROMES OF THE WESTERN PACIFIC
CAPABLE OF SUPPORTING P-3 IARCRAFT

Japan	Misawa AFB Tachikawa AFB NAS Atsugi MCAS Iwakuni
Okinawa	Kadena AFB NAS Naha
Guam	Anderson AFB NAS Agana
Philippines	Clark AFB NAS Cubi Point Naval Station Sangley Point
South Vietnam	Danang Cam Rahn Bay Tan Son Nhut
Taiwan	Tainan

L

FIGURE 1
POSSIBLE COVERAGE OF THE WESTERN PACIFIC
BY PATROL AIRCRAFT BASED IN SELECTED AREAS



III. METHODOLOGY

In the development of the methodology necessary for the optimal allocation of available resources, it will be convenient to assume that an area, A, exists into which it is desired to allocate a specified amount of patrol effort. This desired allocation will be measured in hours and will be assumed to constitute only on-station time. Located around, and within, area A are bases from which the required patrol effort is to be initiated.

To facilitate the development, a rectangular grid will be superimposed upon area A and its supporting bases such that the north-south axis of A is aligned with the vertical axis of the rectangular grid. This grid is to be of sufficient size that all of area A and its supporting bases are enclosed within the borders of the rectangle. A Cartesian coordinate system is then established with the northwest corner of A as the origin, the positive x-axis lying to the east of the origin and the positive y-axis lying to the south of the origin. Distances along the coordinate axes will be measured in nautical miles utilizing the same scale as area A. The rectangular grid will subdivide area A into a number of subareas of equal size. The total number of subareas is the product of the number of columns (n) and the number of rows (m) within the rectangular grid. Assignment of a number i, ranging from one to m to each row, beginning with the uppermost, and a number j, ranging from one to $\underline{n}$ to each column, beginning with the left hand side of A allows each subarea to be denoted by a pair of numbers, (i, j). Figure 2 summarizes the development to this point.

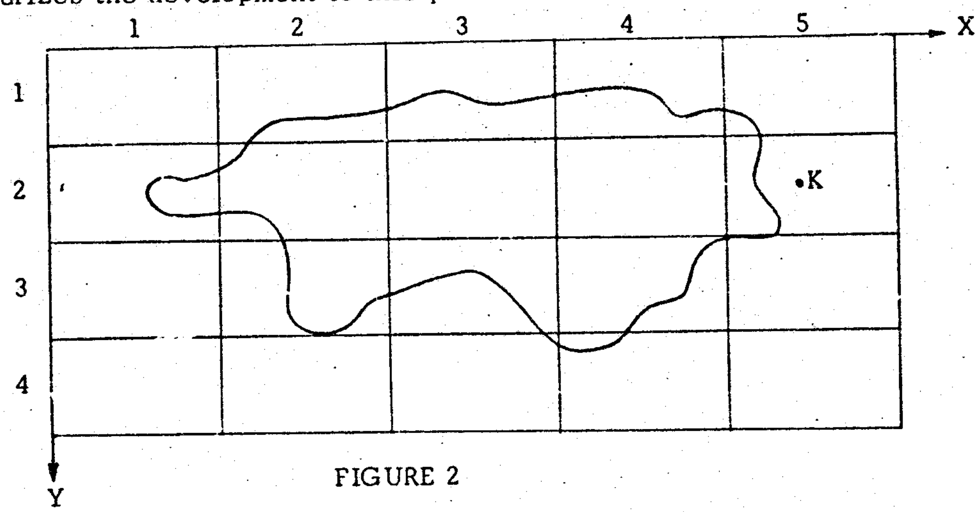


FIGURE 2

AREA A AND GRID OVERLAY

It is now possible to locate any point within the area enclosed by the rectangular grid by either of two methods. For example, the location of the point $\underline{k}$ in Figure 2 may be expressed as (2, 5), indicating that it is within that subarea formed by the intersection of row 2 and column 5; or as (x_k, y_k), which indicates that $\underline{k}$ lies x_k miles to the right of and y_k miles below the origin. By choosing the spacing of the grid lines to be equidistant it is possible to assign a name XL to the length of the side of a subarea parallel to the x-axis and a name YL to the length of the side of a subarea parallel to the y-axis.

For the purposes of this thesis it will be appropriate to assume that any flight designated to operate in a specific area will proceed to the center of that area prior to beginning its on-station period.

The distance, denoted R_{ijk}, between any point $\underline{k}$ and the center of any specific subarea (i,j) may be written as a function of the coordinates of the point $\underline{k}$ and the location of subarea (i,j) in the following form:

$$R_{ijk} = \left[(iYL - \frac{YL}{2} - y_k)^2 + (jXL - \frac{XL}{2} - x_k)^2 \right]^{1/2}$$

If (x_k, y_k) is in fact the location of base $\underline{k}$, then R_{ijk} represents the distance in nautical miles from base $\underline{k}$ to operating area (i,j).

As developed in Appendix A, the available on-station time from base $\underline{k}$, T_{os_k}, in any subarea, per sortie, may be approximated by a linear function of the distance between the base and the operating area, and the average sortie length, in hours, T_k.

$$T_{os_k} = T_k - 0.0052R_{ijk}$$

Further utilizing the results of Appendix A, the maximum desirable operating radius, that which yields an on-station period of at least four hours, is found to be approximately 1350 nautical miles.

A. COSTS

In any problem requiring an optimal allocation of scarce resources it is necessary to evaluate the desirability of each possible alternative. By assigning a weighting factor, measured in dollars, to each variable, it becomes possible to express, in consistent terms, the value associated with each relationship. In the allocation of flight-hours, and hence aircraft, among available sites it is desirable that this factor reflect differences in operating conditions, geographical relationships, and the level of operations.

The system under consideration, that of patrol aircraft and bases, has been in the operating forces for many years. It is not required to consider any costs which might have been associated with any Research and Development, or Investment phase. The annual operating costs, those recurring outlays which are needed to operate and maintain activities in service, the only costs which need to be considered. Large [3] presents the listing shown in Table II, representing a partial breakdown of annual operating expenses.

Examination of those areas listed in Table II discloses several which may be omitted from consideration. PAY AND ALLOWANCES are not directly related to the number of flight hours. Service personnel will be paid whether or not they fly. Similarly, TRAINING and ADMINISTRATIVE AND SUPPORT COSTS must be met even when no flying is performed. Items which do lend themselves to this type of consideration as a direct reflection of flying activity include, FUELS, LUBRICANTS, AND CONSUMABLES as well as some of the MAINTENANCE categories. Consumable items whose usage rates are directly attributable to flying activity include flight clothing, and expendable stores such as sonobuoys, underwater sound signals, and smoke lights. The repair rate for many "Black Box" items is closely related to flight activity. Unfortunately,

TABLE II

ANNUAL OPERATING COSTS

I. EQUIPMENT AND INSTALLATIONS REPLACEMENT

- A. Primary Mission Equipment
- B. Specialized Equipment
- C. Other Equipment
- D. Installations

II. MAINTENANCE

- A. Primary Mission Equipment
- B. Specialized Equipment
- C. Other Equipment
- D. Installations

III. TRAINING

IV. PAY AND ALLOWANCES

V. FUELS, LUBRICANTS, AND OTHER CONSUMABLES

VI. SERVICES AND MISCELLANEOUS

- A. Transportation
- B. Travel
- C. Miscellaneous

VII. ADMINISTRATIVE AND SUPPORT COSTS

the Navy does not have a satisfactory method of assigning a cost to the repair of a particular radio, radar, or other "Black Box" component. It therefore becomes impractical to include repair costs of repairable components in a cost which relates to flying activity.

By comparing the total cost of fuel, lubricants, and consumable items required to operate for a specified period of time with the number of flight-hours flown during the same period it is possible to determine an average cost per flight-hour, denoted CFH. Determining this figure for each location will provide a measure of the cost of operating as influenced by geographical location, operational requirements, and local operating practices.

This figure will now be utilized to develop a costing procedure which can be used for the comparison of selected alternatives. If CFH_k is the cost per flight-hour when flown from base $\underline{k}$, then the cost of one hour of on-station time in any subarea (i, j) that may be reached from base $\underline{k}$ can be determined.

$$\text{COST PER ON-STATION HOUR} = \frac{\text{TOTAL COST OF FLIGHT}}{\text{MAX. NO. OF ON-STATION HOURS}}$$

which yields,

$$C_{ijk} = \frac{(T_k)\ (CFH_k)}{T_k - 0.0052R_{ijk}}$$

or

$$C_{ijk} = \frac{CFH_k}{1 - \frac{0.0052R_{ijk}}{T_k}}$$

C_{ijk} denotes the cost per on-station hour in subarea (i,j) when flown from base k.

B. FORMULATION

Under the assumption of a cost function which has a linear relationship with the on-station hours, the flight-hour allocation problem may be formulated as one which may be solved with the procedures of linear programming. The problem becomes one for which it is desired to fulfill the operational requirements in each subarea at a minimum cost subject to certain restraining conditions expressible as linear equations.

1. Notation

Prior to a formal statement of the problem, notation must be established. If (i,j) denotes a particular operating area and k a specific base, where $i = 1, \ldots, m$, $j = 1, \ldots, n$, and $k = 1, \ldots, p$, then the following definitions will apply:

x_{ijk} number of on-station hours per month allocated to area (i,j) from base k

$x_{m=i,jk}$ number of transit hours per month to area (i,j) from base k in support of x_{ijk}

C_{ijk} cost per on-station hour in area (i, j) when flown from base <u>k</u>

b_{ij} on-station hours per month required in area (i, j)

x_{mis} total hours available per month for training and miscellaneous flying at all bases

a_k flight time in hours per month available from base <u>k</u>

In the flight-hour allocation problem it is necessary to allocate an amount, x_{ijk}, of on-station hours per month from each of <u>p</u> bases among <u>mn</u> operating areas where C_{ijk} is the cost of one hour of on-station time in area (i, j) when flown from base <u>k</u>. Each operating area requires b_{ij} hours of on-station time per month.

The objective function, which represents the cost of providing the required on-station hours, may be expressed as,

$$C = \sum_{k=1}^{p} \sum_{j=1}^{n} \sum_{i=1}^{m} C_{ijk} x_{ijk}.$$

C is now to be minimized subject to the constraints presented below.

2. <u>On-Station Hours</u>

On-station hours allocated to each area from all bases will equal the on-station hours required in each area. This may be written as

$$\sum_{k=1}^{p} x_{ijk} = b_{ij} \quad \text{for } i = 1, \ldots, m \text{ and } j = 1, \ldots, n.$$

3. <u>Transit Hours</u>

In the determination of the total number of flight-hours to be allocated from each base it is desirable to know the number of transit hours necessary to provide the required number of on-station hours. Where R_{ijk} is the distance from base k to area (i,j) the relationship between the on-station time and the transit time may be obtained. From Appendix A, the tradeoff between the on-station time and the transit time on an individual sortie has been shown to be

$$T_{os_k} = T_k - 0.0052R_{ijk}.$$

If x_{ijk} is the number of on-station hours allocated to area (i,j) from base k and T_{os_k} is the average on-station time per sortie, the number of sorties flown may be described as

$$\text{NUMBER OF SORTIES} = \frac{x_{ijk}}{T_{os_k}}.$$

Similarly, if $x_{m+i,jk}$ is the average number of hours of transit time allocated to area (i,j) from base k, the average transit time is

$$T_{tr_k} = 0.0052R_{ijk}.$$

The number of sorties flown is then,

$$\text{NUMBER OF SORTIES} = \frac{x_{m+i,jk}}{T_{tr_k}}.$$

Equating these two equations, the number of on-station hours may be expressed as a function of the number of hours spent in transit.

$$\frac{x_{ijk}}{T_{os_k}} = \frac{x_{m+i,jk}}{0.0052R_{ijk}}$$

As determined previously

$$T_{os_k} = T_k - 0.0052R_{ijk}$$

which is substituted into the equation directly above, yielding, as a constraint;

$$x_{ijk} - x_{m+i,jk}\left[\frac{T_k}{0.0052R_{ijk}} - 1\right] = 0.$$

4. Total Hours Available

The sum of all flight-hours allocated, including training, must equal the total hours available.

$$\sum_{k=1}^{p} \sum_{j=1}^{n} \sum_{i=1}^{2m} x_{ijk} + x_{mis} = A$$

The upper limit of "2m" in the summation over i indicates that both the on-station hours ($i=1,\ldots,m$) and the transit hours ($i=m+1,\ldots,2m$) are to be added.

5. Base Loading

The number of all flight-hours available at each base per month may or may not be known. If the capacity of a base is a significant factor then an upper bound on the number of flight-hours available from base k may exist. If there exists an upper limit to the total available hours at any base k, this restraint may be expressed as:

$$\sum_{j=1}^{n} \sum_{i=1}^{2m} x_{ijk} \leq a_k .$$

C. STATEMENT OF PROBLEM

A complete analytical statement of the flight-hour allocation problem is now possible, to bring together the development of the preceding paragraphs. The problem is then to:

$$\text{Minimize} \quad \sum_{k=1}^{p} \sum_{j=1}^{n} \sum_{i=1}^{m} C_{ijk} X_{ijk}$$

for $i = 1, \ldots, m,\ j = 1, \ldots, n,\ k = 1, \ldots, p$

Subject to,

$$\sum_{k=1}^{p} x_{ijk} = b_{ij}$$

$$x_{ijk} - x_{m+i,jk} \left[\frac{T_k}{0.0052R_{ijk}} - 1 \right] = 0.$$

$$\sum_{k=1}^{p} \sum_{j=1}^{n} \sum_{i=1}^{2m} x_{ijk} + x_{mis} = A$$

$$\sum_{j=1}^{n} \sum_{i=1}^{2m} x_{ijk} \leq a_k$$

D. SOLUTION PROCEDURE

The linear programming problem formulated above is solved by the MPS/360 LP package through the use of a two-phase program in which a routine written in FORTRAN IV translates the necessary input data into a format compatible with the MPS/360 LP requirements. When the transfer of input data has been completed, execution of the MPS/360 LP portion of the program begins. A sample problem is presented in Appendix C and includes a discussion of the output from the MPS/360 LP.

IV. DISCUSSION

A. EXTENSIONS

Other areas to which the methodology presents an immediate solution concern the problem of base denial, the selection of alternate bases, and the problem of an increase in requirements after force levels have been established.

The problem of base denial and the subsequent reallocation of forces may be simulated by removing a base from consideration in the problem formulation. This is readily accomplished by changing the ND entry on the data card for the appropriate base, as shown in Appendix C.

The previously mentioned possibility of base denial raises the question of what alternatives are available if a base is lost. One solution is to reallocate available forces among the remaining bases with the hope of obtaining a feasible solution. Another is to consider the utilization of existing bases not presently supporting patrol forces, or the establishment of new bases.

In any alternative which includes the introduction of a new base or the improvement of existing facilities, care must be taken to ensure that a detailed analysis of all requirements is made. It may evolve that it is less expensive to construct an entire new base than to provide for the incremental adjustments necessary to bring an existing base up to the capability required. Large [3] and WORC [5] have listed many of the items which must be taken into consideration.

One of the primary considerations in any comparison of alternatives is the effectiveness with which the requirements may be met. The methodology presented in this paper may be utilized to assist in this determination. By assigning an expected cost per flight-hour to each location, the alternate bases may be included in the flight-hour allocation procedure. In this manner the effect of each of the alternate sites may be observed. Objective results from the simulation may then be combined with the results of additional comparisons, both subjective and objective, prior to making the final decision.

Requirements for a positive level of training hours or other flight activity may also be included in the solution procedure. If the requirement is one covering all bases, the constraint, $x_{mis} = b_{mis}$, may be placed into the program. To provide for separate requirements at selected bases, the constraint shown above must be broken down for each location, i.e.,

$$x_{mis1} = b_{mis1}, \; x_{mis2} = b_{mis2}.$$

B. ASSUMPTIONS

The formulation of the problem assumes that the total number of flight-hours available will be greater than the total requirement for on-station and transit time. If, however, the situation arises in which the requirements exceed the number of available flight-hours, additional procedures must be instituted. From an academic standpoint the problem may be solved by the establishment of a fictitious base, a_{p+1}, whose available flight-hours are defined as the difference between the hours required and the total hours available.

$$a_{p+1} = \sum_{k=1}^{p+1} \sum_{j=1}^{n} \sum_{i=1}^{2m} x_{ijk} - A$$

Written as a constraint this becomes:

$$\sum_{i=1}^{2m} \sum_{j=1}^{n} \left[\sum_{k=1}^{p+1} x_{ijk} - x_{ijp+1} \right] = A$$

The costs associated with the on-station hours flown between this fictitious base and each operating area should be related to the cost of being unable to furnish the desired coverage of the area. If such a quantitative figure cannot be determined, a cost of zero may be assumed which will then allocate flight-hours on a minimum cost basis to as many areas as possible. In actual practice the problem may be overcome by first comparing the total flight-hours available with those which result from an infeasible solution to the linear programming problem. A subjective decision must then be made as to the necessity of coverage in each subarea, and the amount of coverage desired. By reducing the total requirements a feasible solution to the problem may be obtained.

The manner in which non-feasible base-area combinations are removed from consideration is in need of revision. A more positive method, rather than the assignment of high costs, is necessary. It is possible, in some circumstances, for an undesirable base-area allocation to enter the solution. Such a condition might arise during the solution in the case where the base nearest the area concerned is at its upper bound, if one exists, and all remaining bases are outside the operating radius of the aircraft. In this case, the solution procedure will utilize the only cost available, $999, to achieve a minimum cost allocation.

C. RECOMMENDATIONS FOR IMPROVEMENT AND FURTHER STUDY

The procedure suffers from its dependence upon estimates of operational requirements. While it is possible to obtain objective values based upon past requirements, care must be taken to ensure that the figures are not inflated by subjective estimates of future requirements. An overestimation of these requirements, while providing an excess of available flight-hours for training purposes and unexpected demands, will result in a lower utilization of aircraft and flight crews. The rapid response capability of the P-3 (it is possible to position an aircraft and crew at any point in the Pacific within 24 hours) indicates that operational commanders should position their patrol forces at overseas bases such that the expected level of requirements is met. Unusually heavy and unexpected demands upon the system may be handled by releasing forces from their home port. An alternative method might be a probabilistic interpretation of the flight-hour requirements. This would enable the requirements to be structured such that any chosen level of operations might be handled.

The problem as stated does not take into consideration the possibility of a minimum acceptable level of operation at each base. If a minimum level does exist it may be inserted into the program by selection of an appropriate a_k value and utilization of a greater-than-or-equal-to constraint relationship.

An area which requires considerable study is that of the role played by the training requirements of a deployed squadron. Under the present structure, patrol squadrons are in a state of continual change, with deployed units being made up of both trained and partially trained personnel. This requires a continuing, heavy, training program which often suffers under the weight of operational requirements. Training needs on deployment are filled as the opportunities arise but are continually outpaced by operational demands. It would appear that a more feasible approach to this problem would be the creation of a larger basic unit than the present squadron, which could then deploy a majority of trained personnel, reducing the training requirements at deployed sites to a minimum.

Costs, though they continually play a large role in any problem related to the optimal allocation of resources, are among the more difficult items to identify. The expansion of the concept of a cost per flight-hour to include specific costs for operational, training, and the other types of flying performed, would greatly enhance the capability of the methodology by allowing a more complete breakdown of the requirements.

The assumption of a linear cost function should also be investigated. It is possible that the further division of the cost per flight-hour concept would result in the determination of a non-linear variation between the cost of operating in an area and the time spent in that area. Variables which might enter into the determination of a non-linear relationship include the type of search performed, weather, and search stores expended.

V. SUMMARY

A method has been developed by which force commanders may optimally allocate the patrol forces at their disposal. This is accomplished subject to operational requirements, operating areas, and the forces available. Provided input data defining the location of existing bases, desired coverage of surveillance areas, and available flight-hours, the methodology utilizes the Mathematical Programming System/360 to develop a minimum cost allocation of available forces. The number of aircraft required at each location may be determined by comparing the number of flight hours required within the flying hour capability of the aircraft.

The inputs required for the computer formulation are, the on-station hours required in each subarea, the location of bases under consideration, the flight-hours available at each base, the average sortie length in hours, and the average cost per flight-hour for the aircraft.

The outputs generated are, the total flight-hours required from each base, a complete breakdown of the on-station and transit hours flown from each base, the total time available for training and other missions, and the total cost of providing the on-station coverage required.

The methodology presented in this paper derives a large measure of its usefulness from its inherent flexibility. The sample problem, which consisted of 42 subareas and four bases, required a linear program with 215 row constraints and 337 columns. The MPS/360 LP is capable of solving a linear programming problem with over 4000 row constraints and an unlimited number of columns.

Alternate bases may be included in, or removed from, the solution procedure with a minimal amount of effort, thus providing a rapid, efficient, means of determining the role of each location in the overall picture.

An increase requirement in any area after forces have been deployed may be handled by changing the required on-station time in the area concerned, and adjusting the a_k values of each base to reflect the number of aircraft at each location. The methodology will then determine any necessary reallocation of forces to handle the additional requirements.

APPENDIX A

RELATIONSHIP BETWEEN OPERATING RADIUS AND AVAILABLE ON-STATION TIME

In determining the relationship between the operating radius and the on-station time per sortie it becomes convenient to make the following assumptions regarding the initial configuration of the aircraft:

1. P-3B, takeoff weight of 127,500 pounds.
2. Full fuel load of 59,800 pounds and 300 pounds of water.
3. Outbound flight at 18,000 feet to 22,000 feet altitude.
4. Return flight at 28,000 feet.
5. Zero-fuel weight of 67,400 pounds.
6. Reserve fuel of 8500 pounds.
7. Flight to and from the operating area will be flown according to the maximum range speed schedule as presented in the P-3A/P-3B Natops Handbook.

Based upon the previously stated assumptions and utilizing the material in the P-3 Natops Handbook [9], Figures 3 and 4 can be constructed. Figure 3 depicts the relationship between the gross weight of the aircraft, operating radius, and available on-station time. Figure 4 illustrates the linear relationship which exists between the operating radius and the available on-station time.

A least squares regression analysis of the sample points in Figure 4 results in the relationship

$$T_{os} = 11.2 - 0.0052R$$

between the operating radius and the available on-station hours per sortie. T_{os} is the available on-station hours per sortie, R is the operating radius in nautical miles, and 11.2 is the average sortie length in hours.

Neither Figure 3 nor Figure 4 takes into account the increase in on-station time possible if one or two engines are feathered. The estimates may therefore be considered to be slightly conservative and more useful for planning purposes.

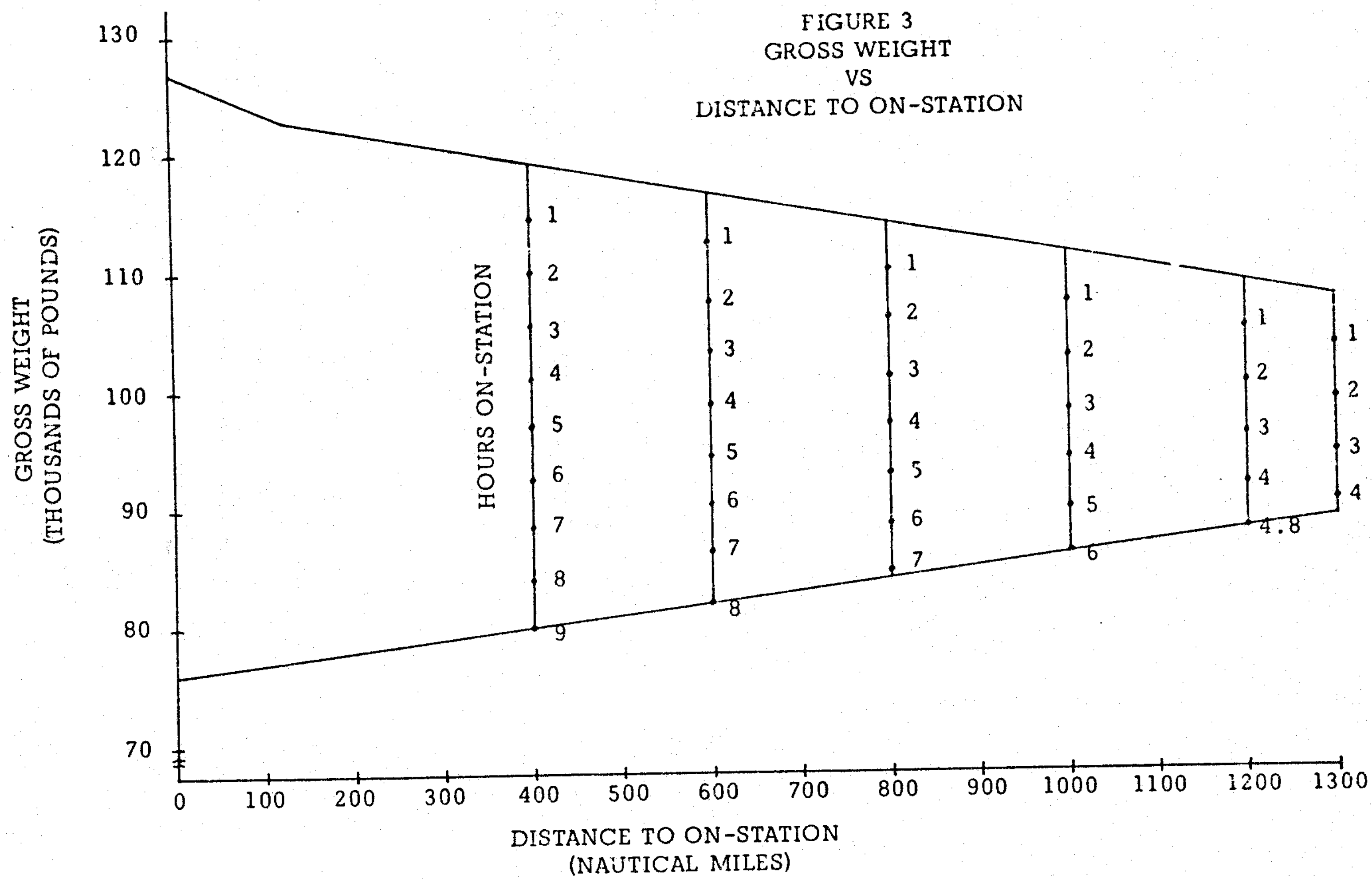
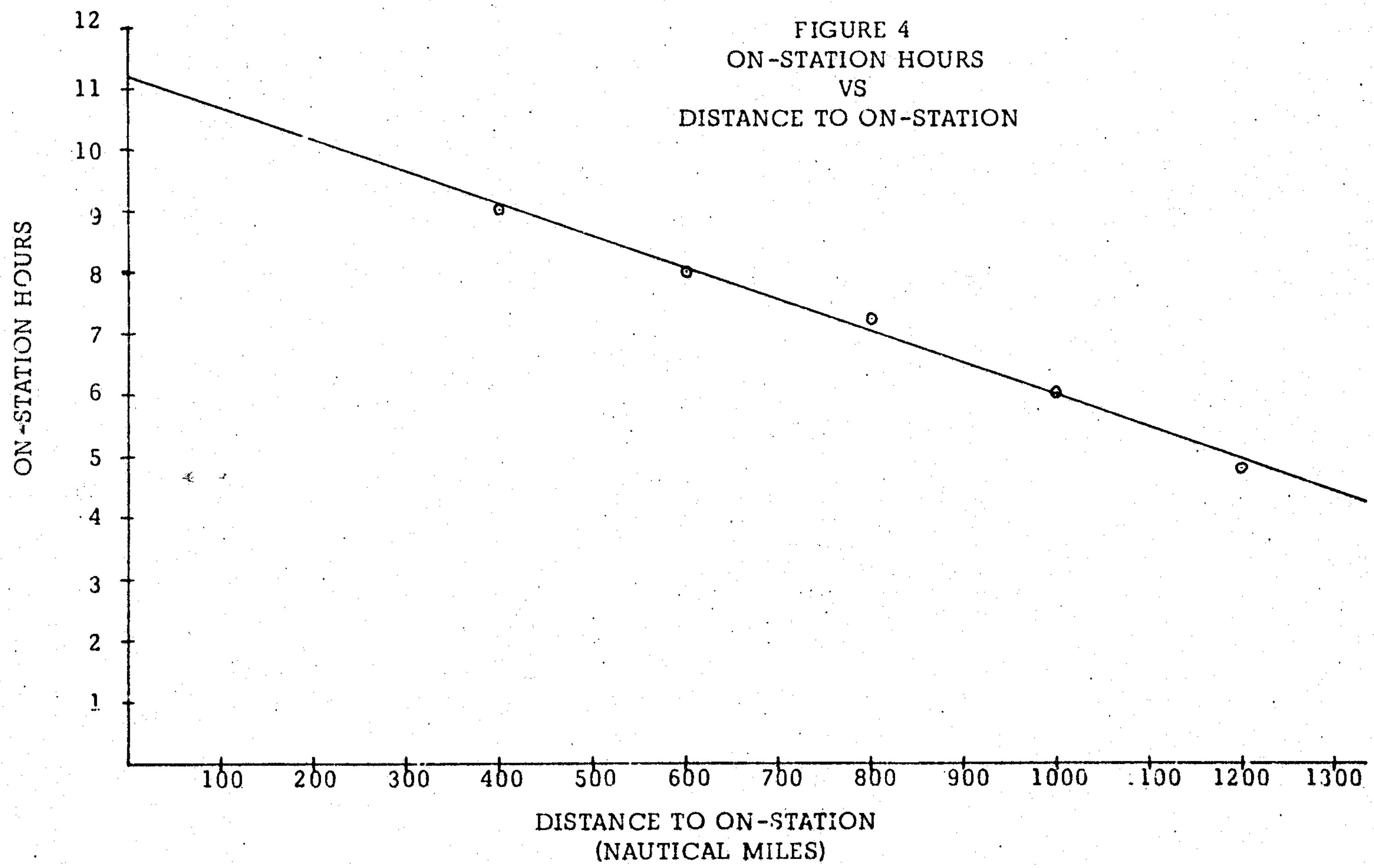


FIGURE 3
GROSS WEIGHT
VS
DISTANCE TO ON-STATION

FIGURE 4
ON-STATION HOURS
VS
DISTANCE TO ON-STATION



APPENDIX B

MAXIMUM FLIGHT HOUR CAPABILITY

The lifetime of an aircraft can be divided into a combination of flying time and ground time. Flying time can be broken down into separate categories to indicate the type of flying performed. Examples of these might be; (1) operational, (2) training, (3) repositioning. For the purpose of determining the maximum flight hour capability all flight time can be treated the same.

Ground time can be divided into the following divisions; (1) Ready-alert and standby, (2) undergoing maintenance, (3) awaiting spares, (4) turn-around time, (5) operationally ready but not flying. In keeping with present Naval terminology (2) and (3) will be referred to as, (2) not operationally ready due to maintenance (NORM) and (3) not operationally ready due to supply (NORS).

The total number of hours available for flight per month per aircraft (average) is 730 hours, as determined by:

$$\text{HOURS PER MONTH} = \frac{24 \text{(hours/day)} \times 365 \text{ (days/year)}}{12 \text{(months/year)}},$$

$$\text{HOURS PER MONTH} = 730 \text{ hours/month}.$$

These 730 hours of available time per month per aircraft may be grouped as follows:

F	FLIGHT HOURS
GA	GROUND ALERT HOURS*
GM	NORM HOURS
GS	NORS HOURS
GT	TURNAROUND HOURS
GO	OPERATIONALLY READY BUT UNSCHEDULED
D	OTHER

* includes ready-alert and standby

For further development of the maximum flying hour capability of the aircraft it will be necessary to determine the number of NORS and NORM hours per flight hour. The number of NORM hours per flight hour for each aircraft may be determined in the following manner. Let K_m be the number of NORM hours per flight hour, then

$$K_m = \frac{GM}{F}.$$

Similarly K_s, the number of NORS hours per flight hour is found to be,

$$K_s = \frac{GS}{F}.$$

The number of available flying hours per month can now be seen to be limited by that time which must be allocated to maintenance, awaiting spare parts and other ground activities. These limitations may be expressed analytically as follows, for each aircraft

$$F + GA + GM + GS + GT + GO + D = 730 \text{ hours/month}$$

and since $GM = K_m F$ and $GS = K_s F$

$$F(1 + K_m + K_s) + GA + GT + GO + D = 730 \text{ hours/month}$$

which yields

$$F_{max} = \frac{730 - (GA + GT + GO + D)}{1 + K_m + K_s} .$$

By minimizing or eliminating the time an aircraft is "operationally ready but not flying" ,(GO), and those unexplained hours, (D), this equation will establish the maximum flying hour capability of the aircraft consistent with current maintenance practices.

APPENDIX C

UTILIZATION

Utilization of the previously developed methodology will now be demonstrated by applications to a sample problem. Following the formulation of the problem; the computer program, preparation of the required input data, and the information contained in the computer output will be presented.

A. SAMPLE PROBLEM

Assume that the operating area is positioned as illustrated in Figure 5. The grid overlay has subdivided the area into 42 subareas, six rows and seven columns. Each subarea is assumed to be 300 miles on a side, yielding a total area covered of 1800 miles by 2100 miles. The four bases shown have the following coordinates, relative to the origin of the grid:

Base	X-coordinate	Y-coordinate
1	1020	1060
2	1140	240
3	2040	480
4	350	1380

The arcs around each base indicate the maximum practical operating radius for that base.

The mission of the patrol forces assigned to these bases will be to provide coastal surveillance coverage of specific areas as indicated

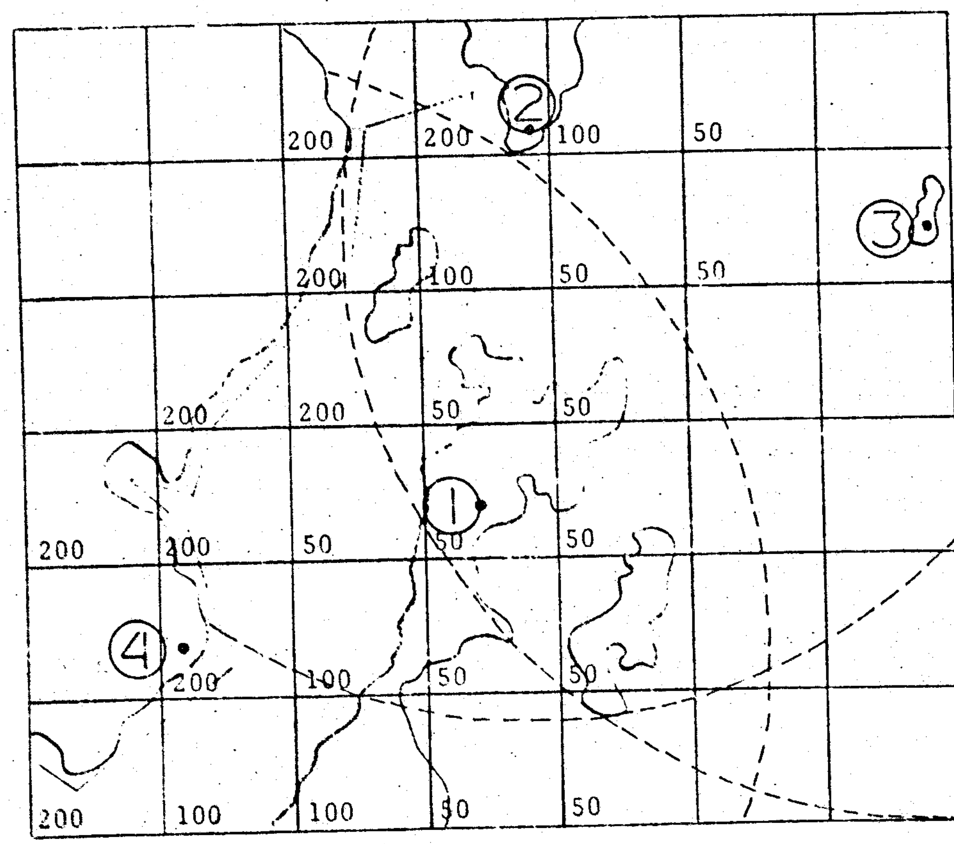


FIGURE 5

PICTORIAL REPRESENTATION OF SAMPLE PROBLEM by the straight line segments in the figure. Additional requirements dictate the need for additional coverage in adjacent areas. From a knowledge of the type of forces available and the operational requirements it is possible to estimate the on-station hours required in each subarea for a specified period. Assume that this has been done for a period of one month and is indicated by the small numbers in each box. If a subarea contains no number indicating the requirement, a requirement does not exist.

The total requirement for on-station hours in the sample problem is then 2900 hours per month. Assume now that the total number of flying hours available in this area per month will be 5500 hours. This figure includes, on-station hours, transit time, training hours and any other flight time.

Base 4 will be assumed to be in an overloaded status and capable of supporting only a limited number of aircraft for patrol purposes. This will be indicated by placing an upper bound on the number of flight-hours available at base 4 of 600 hours per month. The remaining bases, 1,2, and 3 are capable of handling any number of aircraft that might be expected.

Appendix A indicates that the average sortie length utilizing the "profile" concept for maximum aircraft utilization will be approximately 11.2 hours. For the sample problem, assume that this figure will apply to each base.

The last figure required is that of a cost per flight-hour, CFH. This cost may be expressed as its true value, or as a multiple of a base value. For example, if the cost per flight-hour figures for bases 1 through 4 are: $28, $33, $36, $31, they might be also presented as multiples of one of the values, say $28. In this form they would be presented as 1.000, 1.178, 1.391, and 1.107.

B. COMPUTER PROGRAM

The computer program performs the function of translating system requirements into the form required by the MPS/360, then executing the linear program and obtaining an optimal solution to the problem.

The program consists of two parts, a routine written in FORTRAN IV which formulates the input data required for the MPS/360 and places it into storage. An MPS/360 program which retrieves the input data from its storage location, initiates a linear programming solution procedure and determines the optimal allocation.

Inputs to the FORTRAN program are discussed below. After receipt of the input data the FORTRAN program computes the cost per on-station hour utilizing the relationship developed in Appendix A. If the range to any area is found to be greater than the maximum desirable operating radius of 1350 nautical miles, a cost per on-station hour of $999 is assigned to forestall inclusion of a non-feasible base-area combination. If an operating area lies outside the range of all bases considered in a particular problem, the requirement for that area is reduced to zero, removing it from consideration.

The routine then computes the data required by the MPS/360, placing it into storage in the proper sequence. Figure 6 presents an example of the type of data and format necessary for the input to the MPS/360 program.

```
NAME                FLTHRS
ROWS
 N   COST
 E   R1
 .   .
 .   .
 .   .
 E   R25
 G   R26
 L   R27
COLUMNS
 X111               COST          27.63
 X111               R1             1.00
 X111               R16            1.00
   .                  .              .
   .                  .              .
   .                  .              .
 X353               COST          13.26
 X353               R1             1.00
 X353               R26            -.63
RHS
     B              R1           100.00
     B              R2           200.00
     .                .              .
     .                .              .
     .                .              .
     B              R25         6000.00
ENDATA
```

SAMPLE INPUT DATA FOR MPS/360

FIGURE 6

The first card contains the data set name, FLTHRS, and the last card, ENDATA, signifies the end of the data set. ROW cards specify the name to be assigned to the rows of the linear programming matrix, as well as the type of constraint (equality, inequality, or no constraint) represented by the row. COLUMN cards specify the name to be assigned to the columns in the linear programming matrix, and define, in terms of column vectors, the actual values of the matrix elements. RHS cards are used to specify the name of the right-hand-side constraint vector. They are also used to define, in terms of column vectors, the values of these elements. Referring to Figure 6, the following interpretations are made. In the ROWS section, "N   COST" indicates that this is the row corresponding to the objective function of the problem and does not have a constraint. "E   R1" signifies that row R1 is an equality constraint while for row R26 the constraint relationship is greater-than or equal-to. If the only elements in row R1 are found in columns X111, X353, and B, the first equation may be written as

$$X111 + X353 = 100.00.$$

The remaining constraint equations to the problem are formulated in a similar manner.

When the transfer of input data into storage has been completed, execution of the MPS/360 LP portion of the program begins. MPS/360 is composed of a set of procedures, a subset of which deals only with linear programming. The method of solution of the linear programming problem is the ordered execution of a series of these procedures. The user decides upon the method of solution and conveys this to the MPS/360 in the form of the MPS/360 control language. Figure 7 presents the control language program utilized for the solution of the flight-hour allocation problem.

```
PROGRAM
INITIALZ
MOVE(XPBNAME,'PBFILE')
MOVE(XDATA,'FLTHRS')
MOVE(XOBJ,'COST')
MOVE(XRHS,'B')
CONVERT
CRASH
PRIMAL
SOLUTION
EXIT
PEND
```

CONTROL LANGUAGE PROGRAM

FIGURE 7

Complete information regarding the MPS/360 is available in Mathematical Programming System/360, (360-CO-14X) Linear and Separable Programming - Users Manual [4].

C. INPUT DATA

Required data for the solution to the flight-hour problem is of three types:

1. Information regarding the size of the area involved.

2. Flight-hour requirements for each subarea.

3. Base locations, costs per flight-hour at each base, and base utilization.

The data deck is made up of cards containing the above information in the order presented.

1. Area

The first card of the data deck contains six numbers which relate to the number of rows and columns which make up the grid overlay, the number of bases in the area, the length of the sides of each subarea, and the total flight-hours available. This information is conveyed to the program by the following two cards which specify the order and the format of the data.

```
    READ (5,102) M,N,P,XL,YL,AVAIL
102 FORMAT (3I5,3F10.0)
```

For the sample problem, the input data for this section will appear as shown below, with the figures (x) indicating the column in which the first figure is placed.

6	7	4	300.	300.	5500.
(5)	(10)	(15)	(18)	(28)	(38)

2. Flight-Hour Requirements

The flight-hour requirements will be read into the program in an array of the same dimensions as the grid, utilizing the cards presented below.

```
    READ (5,100) ( (B(I,J),J=1,N),I=1,M)
100 FORMAT (6F10.0)
```

The sample problem will appear as follows, each line referring to a separate card.

		200.	200.	100.	50.
			200.	100.	50.
50.			200.	200.	50.
50.			200.	200.	50.
50.	50.				200.
100.	50.	50.			200.
100.	100.	50.	50.		
(1)	(11)	(21)	(31)	(41)	(51)

3. Base Information

The last group of data cards specifies information about each base in the area. The cards;

```
    READ (5,101) (A(I),T(I),CFH(I),X(I),Y(I),ND(I),
   1I=1,P)
101 FORMAT(5F10.0,I2)
```

convey this information to the program. A(I) is a number which corresponds to the maximum number of flight-hours per month that a particular base is capable of supporting. If there is no expected limit this number will be zero. T(I) is the average sortie length, while CFH(I) is the cost per flight-hour. The cost per flight-hour may be represented in either of the two forms mentioned earlier but consistency must be maintained within the program. X(I) and Y(I) correspond to the location of each base. The last figure, ND(I), represents base utilization and may be either zero or one. If a base is to be utilized in the solution procedure the number will be one, if the base is not to be utilized, zero will be used.

Returning to the sample problem, the last section of the data deck will consist of the cards shown below.

0.0	11.2	28.	1020.	1060.	1
0.0	11.2	33.	1140.	240.	1
0.0	11.2	39.	2040.	480.	1
600.	11.2	31.	350.	1380.	1
(1)	(11)	(21)	(31)	(41)	(52)

D. OUTPUT INTERPRETATION

Figure 8 represents a reproduction of several segments of the sample program output. The cost of supplying the required number of operational hours is found in the "ACTIVITY" column under the heading "SOLUTION (OPTIMAL)" to be $108757.55. The "(OPTIMAL)" indicates that an optimal solution was reached. Other possible results are "(NON-OPTIMAL)" and "(INFEASIBLE)." The next section, "SECTION 1 - ROWS," contains the activity levels of each row in the optimal solution. Rows R1 through R42 indicate the operational requirements in each subarea. Row R211 specifies the total number of hours available, while rows R212 through R215 indicate the total flight hours required at each base. The first line following the hours available corresponds to base 1, the second to base 2, and so on. The final section, "SECTION 2 - COLUMNS," provides a complete breakdown of the operational and transit hours flown between each area and each base. For example, column X321 indicates that base 1 is allocating 200.00 hours of on-station time per month to area (3,2) and column X1021 shows

SOLUTION (OPTIMAL)
TIME = 3.20 MINS. ITERATION NUMBER 231

...NAME...	...ACTIVITY...	DEFINED AS
FUNCTIONAL	108757.54745	COST
RESTRAINTS		B

SECTION 1 - ROWS

NUMBER	...ROW...	AT	...ACTIVITY...
1	COST	BS	108757.54745
2	R1	EQ	.
3	R2	EQ	.
4	R3	EQ	200.00
5	R4	EQ	200.00
.	.	.	.
.	.	.	.
212	R211	EQ	5500.00
213	R212	BS	2191.00
214	R213	BS	872.98
215	R214	BS	.
216	R215	EQ	600.00

SECTION 2 - COLUMNS

NUMBER	.COLUMN.	AT	...ACTIVITY...
232	X321	BS	200.00
233	X331	BS	200.00
.	.	.	.
.	.	.	.
407	X1021	BS	71.99
408	X1031	BS	7.17
.	.	.	.
.	.	.	.
553	XMIS	BS	1836.01

SAMPLE COMPUTER OUTPUT

FIGURE 8

that 71.99 hours of transit time are necessary to provide the 200.00 hours of on-station time required in area (3,2). Since the index of transit time requirements runs from i = m+1, ..., 2m, column X1021 refers to the transit time to area (3,2) from base 1. The last entry in "SECTION 2 - COLUMNS," contains the total hours available for other activities. In the sample problem the value of XMIS is 1836.0 hours.

COMPUTER OUTPUT

```
SCLUTICN    (UPTIMAL)
TIME =   1.25 MINS. ITERATICN NUMBER =    231

        ...NAME...      ...ACTIVITY...       DEFINED AS

        FUNCTIONAL      108757.54745         COST
        RESTRAINTS                           B
```

SECTION 1 - ROWS

	NUMBER	...ROW..	AT	...ACTIVITY...
	1	COST	BS	108757.54745
	2	R1	EQ	.
	3	R2	EQ	.
	4	R3	EQ	200.00000
	5	R4	EQ	200.00000
	6	R5	EQ	100.00000
	7	R6	EQ	50.00000
	8	R7	EQ	.
	9	R8	EQ	.
	10	R9	EQ	.
	11	R10	EQ	200.00000
	12	R11	EQ	100.00000
	13	R12	EQ	50.00000
	14	R13	EQ	50.00000
	15	R14	EQ	.
	16	R15	EQ	.
	17	R16	EQ	200.00000
	18	R17	EQ	200.00000
	19	R18	EQ	50.00000
	20	R19	EQ	50.00000
	21	R20	EQ	.
	22	R21	EQ	.
	23	R22	EQ	200.00000
	24	R23	EQ	200.00000
	25	R24	EQ	50.00000
	26	R25	EQ	50.00000
	27	R26	EQ	50.00000
	28	R27	EQ	.
	29	R28	EQ	.
	30	R29	EQ	.
	31	R30	EQ	200.00000
	32	R31	EQ	100.00000
	33	R32	EQ	50.00000
	34	R33	EQ	50.00000
	35	R34	EQ	.
	36	R35	EQ	.
	37	R36	EQ	200.00000
	38	R37	EQ	100.00000
	39	R38	EQ	100.00000
	40	R39	EQ	50.00000
	41	R40	EQ	50.00000
	42	R41	EQ	.
	43	R42	EQ	.
	44	R43	EQ	.
A	45	R44	EQ	.
A	46	R45	EQ	.
A	47	R46	EQ	.
	48	R47	EQ	.
	49	R48	EQ	.

	NUMBER	...ROW..	AT	...ACTIVITY...
	203	R202	EQ	.
	204	R203	EQ	.
	205	R204	EQ	.
	206	R205	EQ	.
	207	R206	EQ	.
	208	R207	EQ	.
	209	R208	EQ	.
	210	R209	EQ	.
	211	R210	EQ	.
A	212	R211	EQ	5500.00000
	213	R212	BS	2191.00007
	214	R213	BS	872.98309
	215	R214	BS	.
	216	R215	LL	600.00000

SECTION 2 - COLUMNS

NUMBER	.COLUMN.	AT	...ACTIVITY...
217	X111	BS	.
218	X121	LL	.
219	X131	LL	.
220	X141	LL	.
221	X151	BS	.
222	X161	BS	.
223	X171	BS	.
224	X211	BS	.
225	X221	BS	.
226	X231	BS	200.00000
227	X241	BS	.
228	X251	BS	.
229	X261	LL	.
230	X271	BS	.
231	X311	BS	.
232	X321	BS	200.00000
233	X331	BS	200.00000
234	X341	BS	50.00000
235	X351	BS	50.00000
236	X361	BS	.
237	X371	LL	.
238	X411	BS	.
239	X421	BS	200.00000
240	X431	BS	50.00000
241	X441	BS	50.00000
242	X451	BS	50.00000
243	X461	BS	.
244	X471	BS	.
245	X511	LL	.
246	X521	BS	196.23895
247	X531	BS	100.00000
248	X541	BS	50.00000
249	X551	BS	50.00000
250	X561	BS	.
251	X571	BS	.
252	X611	LL	.
253	X621	BS	.
254	X631	BS	100.00000
255	X641	BS	50.00000
256	X651	BS	50.00000
257	X661	BS	.
258	X671	BS	.
259	X112	BS	.
260	X122	BS	.
261	X132	BS	200.00000
262	X142	BS	200.00000
263	X152	BS	100.00000
264	X162	BS	50.00000
265	X172	BS	.

NUMBER	.COLUMN.	AT	...ACTIVITY...
266	X212	BS	.
267	X222	BS	.
268	X232	BS	.
269	X242	BS	100.00000
270	X252	BS	50.00000
271	X262	BS	50.00000
272	X272	BS	.
273	X312	BS	.
274	X322	BS	.
275	X332	BS	.
276	X342	BS	.
277	X352	BS	.
278	X362	BS	.
279	X372	BS	.
280	X412	BS	.
281	X422	BS	.
282	X432	BS	.
283	X442	BS	.
284	X452	BS	.
285	X462	BS	.
286	X472	BS	.
287	X512	BS	.
288	X522	BS	.
289	X532	BS	.
290	X542	BS	.
291	X552	BS	.
292	X562	BS	.
293	X572	BS	.
294	X612	BS	.
295	X622	BS	.
296	X632	BS	.
297	X642	BS	.
298	X652	BS	.
299	X662	BS	.
300	X672	BS	.
301	X113	BS	.
302	X123	BS	.
303	X133	BS	.
304	X143	BS	.
305	X153	BS	.
306	X163	BS	.
307	X173	BS	.
308	X213	BS	.
309	X223	BS	.
310	X233	BS	.
311	X243	BS	.
312	X253	BS	.
313	X263	BS	.
314	X273	BS	.
315	X313	BS	.
316	X323	BS	.

NUMBER	.COLUMN.	AI	...ACTIVITY...
317	X333	BS	.
318	X343	BS	.
319	X353	BS	.
320	X363	BS	.
321	X373	BS	.
322	X413	BS	.
323	X423	BS	.
324	X433	BS	.
325	X443	BS	.
326	X453	BS	.
327	X463	BS	.
328	X473	BS	.
329	X513	BS	.
330	X523	BS	.
331	X533	BS	.
332	X543	BS	.
333	X553	BS	.
334	X563	BS	.
335	X573	BS	.
336	X613	LL	.
337	X623	BS	.
338	X633	BS	.
339	X643	BS	.
340	X653	BS	.
341	X663	BS	.
342	X673	BS	.
343	X114	BS	.
344	X124	BS	.
345	X134	BS	.
346	X144	BS	.
347	X154	BS	.
348	X164	BS	.
349	X174	BS	.
350	X214	BS	.
351	X224	BS	.
352	X234	BS	.
353	X244	BS	.
354	X254	BS	.
355	X264	BS	.
356	X274	BS	.
357	X314	BS	.
358	X324	BS	.
359	X334	BS	.
360	X344	BS	.
361	X354	BS	.
362	X364	BS	.
363	X374	BS	.
364	X414	BS	200.00000
365	X424	BS	.
366	X434	BS	.
367	X444	BS	.

NUMBER	.COLUMN.	AT	...ACTIVITY...
368	X454	BS	.
369	X464	BS	.
370	X474	BS	.
371	X514	BS	.
372	X524	BS	3.76105
373	X534	BS	.
374	X544	BS	.
375	X554	BS	.
376	X564	BS	.
377	X574	BS	.
378	X614	BS	200.00000
379	X624	BS	100.00000
380	X634	BS	.
381	X644	BS	.
382	X654	BS	.
383	X664	BS	.
384	X674	BS	.
385	X711	LL	.
386	X721	BS	.
387	X731	BS	.
388	X741	BS	.
389	X751	LL	.
390	X761	LL	.
391	X771	LL	.
392	X811	BS	.
393	X821	BS	.
394	X831	BS	89.72633
395	X841	LL	.
396	X851	LL	.
397	X861	BS	.
398	X871	LL	.
399	X911	BS	.
400	X921	BS	86.20690
401	X931	BS	47.18094
402	X941	BS	8.45166
403	X951	BS	13.30849
404	X961	BS	.
405	X971	BS	.
406	X1011	LL	.
407	X1021	BS	71.99424
408	X1031	BS	7.17154
409	X1041	BS	.74503
410	X1051	BS	9.05141
411	X1061	BS	.
412	X1071	BS	.
413	X1111	BS	.
414	X1121	BS	82.87118
415	X1131	BS	22.54283
416	X1141	BS	7.82718
417	X1151	BS	12.81066
418	X1161	BS	.

NUMBER	.COLUMN.	AT	...ACTIVITY...
419	X1171	BS	.
420	X1211	BS	.
421	X1221	LL	.
422	X1231	BS	43.10345
423	X1241	BS	18.89645
424	X1251	BS	22.87283
425	X1261	BS	.
426	X1271	BS	.
427	X712	BS	.
428	X722	BS	.
429	X732	BS	45.65168
430	X742	BS	12.56124
431	X752	BS	11.86662
432	X762	BS	15.82779
433	X772	LL	.
434	X[illegible]2	LL	.
435	X822	LL	.
436	X832	LL	.
437	X842	BS	11.86662
438	X852	BS	7.99744
439	X862	BS	17.21170
440	X872	LL	.
441	X912	LL	.
442	X922	LL	.
443	X932	LL	.
444	X942	LL	.
445	X952	LL	.
446	X962	LL	.
447	X972	LL	.
448	X1012	LL	.
449	X1022	LL	.
450	X1032	LL	.
451	X1042	LL	.
452	X1052	LL	.
453	X1062	LL	.
454	X1072	LL	.
455	X1112	LL	.
456	X1122	LL	.
457	X1132	LL	.
458	X1142	LL	.
459	X1152	LL	.
460	X1162	LL	.
461	X1172	LL	.
462	X1212	LL	.
463	X1222	LL	.
464	X1232	LL	.
465	X1242	LL	.
466	X1252	LL	.
467	X1262	LL	.
468	X1272	LL	.
469	X713	LL	.

NUMBER	.COLUMN.	AT	...ACTIVITY...
521	X844	LL	.
522	X854	LL	.
523	X864	LL	.
524	X874	LL	.
525	X914	LL	.
526	X924	LL	.
527	X934	LL	.
528	X944	LL	.
529	X954	LL	.
530	X964	LL	.
531	X974	LL	.
532	X1014	BS	43.64906
533	X1024	LL	.
534	X1034	LL	.
535	X1044	LL	.
536	X1054	LL	.
537	X1064	LL	.
538	X1074	LL	.
539	X1114	BS	.
540	X1124	BS	.19160
541	X1134	LL	.
542	X1144	LL	.
543	X1154	LL	.
544	X1164	LL	.
545	X1174	LL	.
546	X1214	BS	36.96858
547	X1224	BS	15.42972
548	X1234	LL	.
549	X1244	LL	.
550	X1254	LL	.
551	X1264	LL	.
552	X1274	LL	.
553	XMIS	BS	1836.01684

COMPUTER PROGRAM

```
       PROGRAM COMPUTES INPUT DATA REQUIRED
       FOR THE SOLUTION OF THE FLIGHT HOUR
       ALLOCATION PROBLEM AND PRESENTS THE DATA
       IN A FORMAT COMPATIBLE WITH THE INPUT
       REQUIREMENTS FOR THE IBM SYSTEM 360
       MATHEMATICAL PROGRAMMING SYSTEM.

       VARIABLE NAMES AND PROGRAM INPUTS

    M.........NUMBER OF ROWS OF REQUIREMENTS ARRAY
    N.........NUMBER OF COLUMNS OF REQUIREMENTS ARRAY
    P.........NUMBER OF BASES
    XL........LENGTH OF HORIZONTAL SIDE OF SUBAREA
    YL........LENGTH OF VERTICAL SIDE OF SUBAREA
    B(I,J)....FLIGHT HOUR REQUIREMENTS PER AREA
    X(K)......X-COORDINATE OF BASE LOCATION
    Y(K)......Y-COORDINATE OF BASE LOCATION
    AVAIL.....TOTAL FLIGHT HOURS AVAILABLE
    T(K)......AVERAGE SORTIE LENGTH FROM EACH BASE
    CFH(K)....COST PER FLIGHT HOUR FROM EACH BASE
    ND(K).....DENOTES IF BASE IS TO BE UTILIZED

100  FORMAT(8F10.0)
101  FORMAT(5F10.0,I2)
102  FORMAT(3I5,3F10.0)
200  FORMAT('NAME',T15,'FLTHRS',T80,'A'/'ROWS',T80,'A'/T2,
    1'N',T5,'COST',T80,'A')
201  FORMAT(T2,'E',T5,'R',I3,T80,'A')
202  FORMAT('COLUMNS',T80,'A')
203  FORMAT(T2,'G',T5,'R',I3,T80,'A')
204  FORMAT(T5,'X',3I2,T15,'COST',T25,F10.3,T80,'A')
205  FORMAT(T5,'X',3I2,T15,'R',I3,T31,'1.00',T80,'A')
206  FORMAT(T5,'X',3I2,T15,'R',I3,T25,F10.3,T80,'A')
207  FORMAT('RHS',T80,'A')
208  FORMAT(T5,'B',T15,'R',I3,T25,F10.3,T80,'A')
209  FORMAT('ENDATA',T80,'A')
210  FORMAT(T2,'L',T5,'R',I3,T80,'A')
211  FORMAT(T5,'XNIS',T15,'R',I3,T31,'1.00',T80,'A')
2010 FORMAT(T2,'F',T5,'R',I2,T80,'A')
2080 FORMAT(T5,'B',T15,'R',I2,T25,F10.3,T80,'A')
     DIMENSION B(20,20),X(10),Y(10),R(20,20,10),T(10),
    1CFH(10),C(20,20,10),A(10),ND(10)

     INTEGER P,PX
     DATA ZERO,PX/0.0,0/
     READ(5,102) M,N,P,XL,YL,AVAIL
     READ (5,100) ((B(I,J),J=1,N),I=1,M)
     READ (5,101) (A(I),T(I),CFH(I),X(I),Y(I),ND(I),I=1,P)
     I1=M*N*(P+1)+1
     IROW=I1+P
     M1=M+1
     M2=2*M
     DO 10 K=1,P
     DO 10 I=1,M
     DO 10 J=1,N
     X1=J
     Y1=I
     RY=ABS(Y1*YL-(YL/2.0)-Y(K))
     RX=ABS(X1*XL-(XL/2.0)-X(K))
     R(I,J,K)=SQRT(RX**2.+RY**2.)
C        COMPUTE COST PER ON STATION HOUR

     C(I,J,K)=CFH(K)/(1.-(0.0052*R(I,J,K)/T(K)))
```

```
C               IF RANGE TO OPERATING AREA IS GREATER
C               THAN 1350 NAUTICAL MILES ASSIGN A COST
C               OF $999.00 PER HOUR
      DO 18 I=1,P
   18 RX=ND(I)+RX
      DO 12 I=1,M
      DO 12 J=1,N
      L=0
      DO 11 K=1,P
      RX=R(I,J,K)
C               IF AN AREA CANNOT BE REACHED FROM ANY
C               BASE REDUCE ITS REQUIREMENT TO ZERO
   11 IF(RX.GT.1350.) L=L+1
   12 IF(L.GE.RX) B(I,J)=0.0
      WRITE(8,200)
C               COMPUTE DATA FOR "ROWS" SECTION
      DO 19 I=1,IROW
      IF(I1-I) 13,17,17
   13 K=I-I1
      IF(A(K)) 14,14,16
   14 IF(ND(K).EQ.0) GO TO 17
   15 WRITE(8,203) I
      GO TO 19
   16 WRITE(8,210) I
      GO TO 19
   17 IF(I-100) 170,171,171
  170 WRITE(8,2010) I
      GO TO 19
  171 WRITE(8,201) I
   19 CONTINUE
C               COMPUTE DATA FOR "COLUMNS" SECTION
      WRITE(8,202)
      DO 20 K=1,P
      DO 20 I=1,M
      DO 20 J=1,N
      I2=I1+K
      I205=(I-1)*N+J
      I205A=K*M*N+(I-1)*N+J
      WRITE(8,204) I,J,K,C(I,J,K)
      WRITE(8,205) I,J,K,I205
      WRITE(8,205) I,J,K,I205A
      WRITE(8,205) I,J,K,I1
   20 WRITE(8,205) I,J,K,I2
      DO 30 K=1,P
      DO 30 I=M1,M2
      DO 30 J=1,N
      I2=I1+K
      I205A=K*M*N+(I-M-1)*N+J
      IM=I-M
      CX=1.00-(T(K)/(0.0052*R(IM,J,K)))
      WRITE(8,206) I,J,K,I205A,CX
      WRITE(8,205) I,J,K,I1
   30 WRITE(8,205) I,J,K,I2
      WRITE(8,211) I1
C               COMPUTE DATA FOR "RHS" SECTION
      WRITE(8,207)
      DO 40 I=1,M
      DO 40 J=1,N
      I207=(I-1)*N+J
      IF(I207-100) 400,401,401
  400 WRITE(8,2080) I207,B(I,J)
      GO TO 40
  401 WRITE(8,208) I207,B(I,J)
```

```
   40 CONTINUE
      DO 50 K=1,P
      DO 50 I=1,M
      DO 50 J=1,N
      I207A=K*M*N+(I-1)*N+J
      IF(I207A-100) 500,501,501
  500 WRITE(8,2090) I207A,ZERO
      GO TO 50
  501 WRITE(8,208) I207A,ZERO
   50 CONTINUE
      WRITE(8,208) I1,AVAIL
      DO 70 K=1,P
      I2=I1+K
   70 WRITE(8,208) I2,A(K)
      WRITE(8,209)
      STOP
      END
```

```
//GO.FT08F001 DD UNIT=SYSDA,DSN=&MPS,DCB=(RECFM=FB,BLKSIZE=8
//         LRECL=80),DISP=(NEW,PASS),SPACE=(TRK,(30,2),RLSE)
//GO.SYSIN DD *
```

INPUT DATA DECK

6	7 4	300.	300.	5500.	
0.0	0.0	200.	200.	100.	50.
0.0	0.0	0.0	200.	100.	50.
50.	0.0	0.0	200.	200.	50.
50.	0.0	0.0	200.	200.	50.
50.	50.	0.0	0.0	0.0	200.
100.	50.	50.	0.0	0.0	200.
100.	100.	50.	50.	0.0	0.0
0.0	11.2	28.	1020.	1060.	1
0.0	11.2	33.	1140.	240.	1
0.0	11.2	39.	2040.	480.	1
600.	11.2	31.	350.	1380.	1

```
//S2   EXEC LINPROG
//MPS1.SAVE DD DSN=*.S1.GO.FT08F001,DISP=(OLD,PASS)
//MPS1.SYSIN DD *
```

CONTROL LANGUAGE PROGRAM FOR MPS/360

```
PROGRAM
INITIALZ
MOVE(XPBNAME,'PBFILE')
MOVE(XDATA,'FLTHRS')
MOVE(XOBJ,'COST')
MOVE(XRHS,'B')
CONVERT
SETUP
CRASH
PRIMAL
SOLUTION
EXIT
PEND
```

```
//MPS2.SYSPRINT DD SPACE=(CYL,6)
//MPS2.SYSIN DD  DSN=*.S1.GO.FT08F001.DISP=(OLD,DELETE)
```

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13. ABSTRACT

A methodology is developed which determines the optimal allocation of patrol forces among selected deployment sites. The procedure uses a linear programming algorithm which minimizes a linear cost function, subject to restraining equations representing the total hours available, the relationship between on-station and transit hours, and base loading. A computer program is presented which translates input data into the format required by the IBM Mathematical Programming System/360 for the problem solution. The methodology can be utilized to determine the allocation of forces among selected bases, reallocation of forces when a base or bases must be removed from consideration, and the effect of utilizing additional bases.

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